[13.40] Let V be an *n*-dimensional vector space,  the tensor product of V with itself, and tensor. Let

 be the symmetric part

and

 be the antisymmetric part.

Define

 and .

Then

 and .

Solution.

Let  be the basis for V where . Set

.

By definition,  is a basis for V, and it has *n*2 terms. Observe that  and . So, we define

 and .

 has  terms with *a* ≤ *b* and  has  terms with *a* < *b*.

Note: The reason for defining  and  with *a* ≤ *b* is that  and , so terms with *b* > *a* are not independent from the others. As defined, each of  and  consists of linearly independent vectors, so

dim span  and dim span .

We proceed to show that  and , which completes the problem.

First, since , we have that .

Next, we claim that :

. Since each  is symmetric, *Q* is symmetric. That is, .

Similarly, if  then *Q* is antisymmetric, or .

Finally, 

 ✔

Example with *n* = 2:

Let  and . Then  is a basis for V. Let , , , and . Then

 is a basis for . Observe that  and . These are 2 elements of . Note that  so they do not contribute to . The other term in  is . The only term in  is  (which equals ). Thus dim  3 and dim .